

Nonrelativistic Quantum Mechanics with Spin in the Framework of a Classical Subquantum Kinetics

G. Kaniadakis

Dipartimento di Fisica and Istituto Nazionale di Fisica della Materia,

Politecnico di Torino,

Corso Duca degli Abruzzi 24, 10129 Torino, Italy

E-mail address: kaniadakis@polito.it

(Dated: January 29, 2003)

Recently it has been shown that the spinless one particle quantum mechanics can be obtained in the framework of entirely classical subquantum kinetics. In the present paper we argue that, within the same scheme and without any extra assumption, it is possible to obtain both the non relativistic quantum mechanics with spin, in the presence of an arbitrary external electromagnetic field, as well as the nonlinear quantum mechanics.

PACS numbers: 03.65.Ta, 05.20.Dd

KEY WORDS: monads, subquantum physics, quantum potential, foundations of quantum mechanics

I. INTRODUCTION

Certainly one of the most intricate questions in quantum mechanics concerns the origin of quantum potential [1, 2, 3, 4]. Recently in ref. [4] the quantum potential has been obtained in the framework of classical many body physics without invoking any epistemological principle. This unexpected result gives rise to the hope that quantum physics can be obtained in a self consistent scheme of an entirely classical many body physics. Indeed, in the same reference within this classical framework, the main features of quantum mechanics (i.e. the probabilistic nature of the quantum description, the Schrödinger equation, the quantum operators, the Heisenberg uncertainty principle) have been deduced in the case of a spinless particle. It is surprising that this theory contains only one free parameter which is identifiable with the fundamental constant \hbar .

Within this classical scheme the quantum particle turns out to have an internal structure and a spatial dispersion and appears to be composed by N identical point like and interacting subquantum objects, the monads, obeying to the laws of classical physics. The statistical ensemble of these N monads is governed in the phase space by a kinetics, which takes into account that, during the point-like collisions, the monad number, momentum and energy are conserved. It is remarkable that simply the projection of the above phase space kinetics, into the physical space, produces a hydrodynamics which leads naturally to the one particle spinless quantum mechanics, when the external force field is conservative.

Clearly this latter condition is instead restrictive. A first question which naturally arises is if it is possible to obtain quantum mechanics for a particle with spin in the presence of an arbitrary time dependent external electromagnetic field [5, 6, 7, 8, 9], within the above N -body classical kinetics. A second question is if it is possible to obtain also the nonlinear quantum mechanics [10], within the same scheme. The answers to these questions, as we will see in the following, are affirmative.

The main purpose of the present work is to show that it is possible to obtain both the non relativistic quantum mechanics with spin as well the nonlinear quantum mechanics, following the general lines of the theory developed in ref. [4], without making any additional assumption.

II. CLASSICAL HYDRODYNAMICS

Let us consider the phase space kinetics of a system of N monads, each of mass μ and charge q . We suppose that the system of total mass $m = N\mu$ and charge $e = Nq$, is immersed in an arbitrary external electromagnetic field derived from the time dependent potential $A^{(ex)} = \left(A_0^{(ex)}, \mathbf{A}^{(ex)} \right)$ by means of

$$\mathbf{E}^{(ex)} = -\nabla A_0^{(ex)} - \frac{1}{c} \frac{\partial \mathbf{A}^{(ex)}}{\partial t} ; \quad \mathbf{B}^{(ex)} = \nabla \times \mathbf{A}^{(ex)} . \quad (1)$$

The distribution function, obeying the normalization condition $\int f d^3v d^3x = N$ of such a system, is governed by the kinetic equation [11, 12]

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{\mu} \left(\mathbf{E}^{(ex)} + \frac{1}{c} \mathbf{v} \times \mathbf{B}^{(ex)} \right) \cdot \nabla_v f = C(f) . \quad (2)$$

The assumption that during the point-like collisions the monad number, momentum and energy are conserved, implies that the three functions $g_1(\mathbf{v}) = 1$, $g_2(\mathbf{v}) = \mathbf{v}$ and $g_3(\mathbf{v}) = \mathbf{v}^2$ are the collision invariants of the system and thus the collision integral $C(f)$ satisfies the conditions: $\int g_j(\mathbf{v}) C(f) d^3v = 0$ with $j = 1, 2, 3$.

In the following we study the projection of the system dynamics in the physical space, where the distribution function is $\rho(t, \mathbf{x}) = \int f d^3v$ and the mean value of a given property $G = G(t, \mathbf{x}, \mathbf{v})$ of the system, in the point \mathbf{x} , is defined as $\langle G \rangle_v = \rho^{-1} \int G f d^3v$. Then, we can introduce the density of current $\mathbf{u} = \langle \mathbf{v} \rangle_v$ and the

symmetric tensor $\sigma_{ij} = \sigma_{ji}$ density of stress

$$\sigma_{jk} = \mu (\langle v_j v_k \rangle_v - \langle v_j \rangle_v \langle v_k \rangle_v) . \quad (3)$$

Multiplying Eq.(2) by the two first collision invariants $g_1(\mathbf{v})$ and $g_2(\mathbf{v})$ and after integration with respect to \mathbf{v} , the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 , \quad (4)$$

and the momentum balance equation

$$\frac{\partial}{\partial t}(\mu \rho u_j) + \frac{\partial}{\partial x_k}(\mu \rho u_j u_k + \rho \sigma_{jk}) - \rho F_j^{(ex)} = 0 , \quad (5)$$

with

$$\mathbf{F}^{(ex)} = q \left(\mathbf{E}^{(ex)} + \frac{1}{c} \mathbf{u} \times \mathbf{B}^{(ex)} \right) , \quad (6)$$

can be obtained respectively [11, 12].

It is important to emphasize that in the Lorentz force acting on a single monad, as we can see from Eq. (2), appears the monad velocity \mathbf{v} , while in the expression of the Lorentz $\mathbf{F}^{(ex)}$ given by Eq. (6), appears \mathbf{u} which describes a collective property of the system. Eq.s (4) and (5) are the hydrodynamic equations for the system, which conserves its monad number $N = \int \rho d^3x$ and behaves as a fluid in the physical space.

After taking into account Eq. (4), we rewrite Eq. (5) in the following Newton-like form

$$\mu \frac{D\mathbf{u}}{Dt} = \mathbf{F}^{(\sigma)} + \mathbf{F}^{(ex)} , \quad (7)$$

where $D/Dt = \partial/\partial t + \mathbf{u} \cdot \nabla$ is the total time (or Lagrangian or substantial) derivative. The hydrodynamic force $\mathbf{F}^{(\sigma)}$, generated, through the stress tensor, from the monad interactions, takes the form

$$F_j^{(\sigma)} = -\frac{1}{\rho} \frac{\partial}{\partial x_k} \rho \sigma_{jk} . \quad (8)$$

We now focus our attention to an interesting property of the force $-\mu D\mathbf{u}/Dt$. It is trivial to verify that this force has a Lorentz-like structure

$$-\mu \frac{D\mathbf{u}}{Dt} = q \left(\mathbf{E}^{(u)} + \frac{1}{c} \mathbf{u} \times \mathbf{B}^{(u)} \right) , \quad (9)$$

where the fields $\mathbf{E}^{(u)}$ and $\mathbf{B}^{(u)}$ can be derived from the potential $A^{(u)}$

$$A^{(u)} = (A_0^{(u)}, \mathbf{A}^{(u)}) = \frac{\mu}{q} \left(\frac{1}{2} \mathbf{u}^2, c\mathbf{u} \right) , \quad (10)$$

according to

$$\mathbf{E}^{(u)} = -\nabla A_0^{(u)} - \frac{1}{c} \frac{\partial \mathbf{A}^{(u)}}{\partial t} ; \quad \mathbf{B}^{(u)} = \nabla \times \mathbf{A}^{(u)} . \quad (11)$$

The Lorentz-like structure of $-\mu D\mathbf{u}/Dt$ is exclusively enforced by the projection mechanism of particle motion from the phase space into the physical space (kinetics \rightarrow hydrodynamics). An immediate consequence of the Lorentz-like structure of $-\mu D\mathbf{u}/Dt$ is that Eq. (7) imposes a Lorentz-like structure also for $\mathbf{F}^{(\sigma)}$:

$$\mathbf{F}^{(\sigma)} = q \left(\mathbf{E}^{(\sigma)} + \frac{1}{c} \mathbf{u} \times \mathbf{B}^{(\sigma)} \right) , \quad (12)$$

where the fields $\mathbf{E}^{(\sigma)}$ and $\mathbf{B}^{(\sigma)}$ can be derived from the potential $A^{(\sigma)} = (A_0^{(\sigma)}, \mathbf{A}^{(\sigma)})$ by means of

$$\mathbf{E}^{(\sigma)} = -\nabla A_0^{(\sigma)} - \frac{1}{c} \frac{\partial \mathbf{A}^{(\sigma)}}{\partial t} ; \quad \mathbf{B}^{(\sigma)} = \nabla \times \mathbf{A}^{(\sigma)} . \quad (13)$$

We observe now that, after introducing the two fields $\mathbf{E} = \mathbf{E}^{(u)} + \mathbf{E}^{(\sigma)} + \mathbf{E}^{(ex)}$ and $\mathbf{B} = \mathbf{B}^{(u)} + \mathbf{B}^{(\sigma)} + \mathbf{B}^{(ex)}$, which can be derived from the potential $A = (A_0, \mathbf{A}) = A^{(u)} + A^{(\sigma)} + A^{(ex)}$, Eq. (7) can be written as

$$\mathbf{E} + \frac{1}{c} \mathbf{u} \times \mathbf{B} = 0 . \quad (14)$$

Of course, Eq. (14) is satisfied if simultaneously $\mathbf{E} = 0$ and $\mathbf{B} = 0$. The condition $\mathbf{B} = 0$ implies $\nabla \times \mathbf{A} = 0$ so that we can write $q\mathbf{A} = c\nabla\mathcal{S}$. From this last relation and the definition of \mathbf{A} we obtain

$$\mathbf{u} = \frac{1}{\mu} \left(\nabla\mathcal{S} - \frac{q}{c} \mathbf{A}^{(ex)} - \frac{q}{c} \mathbf{A}^{(\sigma)} \right) . \quad (15)$$

On the other hand, the condition $\mathbf{E} = 0$ implies the equation $\partial\mathbf{A}/\partial t + c\nabla A_0 = 0$ which, after combined with $q\mathbf{A} = c\nabla\mathcal{S}$, becomes $\partial\mathcal{S}/\partial t + qA_0 = 0$. This last equation, after taking into account the definition of A_0 and Eq. (15), assumes the form

$$\frac{\partial\mathcal{S}}{\partial t} + \frac{1}{2\mu} \left(\nabla\mathcal{S} - \frac{q}{c} \mathbf{A}^{(ex)} - \frac{q}{c} \mathbf{A}^{(\sigma)} \right)^2 + qA_0^{(ex)} + qA_0^{(\sigma)} = 0 . \quad (16)$$

The vorticity [5] of a particle system of total mass m and charge e , immersed in an external electromagnetic field, is defined through $\boldsymbol{\omega} = \nabla \times (m\mathbf{u} + (e/c)\mathbf{A}^{(ex)})$ and results from the stress force $\mathbf{F}^{(\sigma)}$, being $\boldsymbol{\omega} = -(e/c)\nabla \times \mathbf{A}^{(\sigma)} = -(e/c)\mathbf{B}^{(\sigma)}$, as one can verify immediately by considering Eq. (15).

According to ref. [5] we can decompose the stress tensor density as $\sigma_{jk} = \varsigma_{jk} + \nu_{jk}$. The first term is the residual or nonvortical stress tensor density, which is present also when $\boldsymbol{\omega} = 0$. The second term represents the vortical stress tensor density, which originates the vortical flow in the system. From Eq. (8) we have that $\mathbf{F}^{(\sigma)} = \mathbf{F}^{(\varsigma)} + \mathbf{F}^{(\nu)}$. Of course it results $A_0^{(\sigma)} = A_0^{(\varsigma)} + A_0^{(\nu)}$, while we can pose $\mathbf{A}^{(\sigma)} = \mathbf{A}^{(\nu)}$ in order to write $\boldsymbol{\omega} = -(e/c)\nabla \times \mathbf{A}^{(\nu)}$ and then $\boldsymbol{\omega} = -(e/c)\mathbf{B}^{(\nu)}$. Consequently for the nonvortical stress force we have

$$F_j^{(\varsigma)} = -\frac{1}{\rho} \frac{\partial}{\partial x_k} \rho \varsigma_{jk} ; \quad \mathbf{F}^{(\varsigma)} = -q\nabla A_0^{(\varsigma)} . \quad (17)$$

For the vortical stress force we obtain

$$\mathbf{F}_j^{(\nu)} = -\frac{1}{\rho} \frac{\partial}{\partial x_k} \rho \nu_{jk} \quad ; \quad \mathbf{F}^{(\nu)} = q \left(\mathbf{E}^{(\nu)} + \frac{1}{c} \mathbf{u} \times \mathbf{B}^{(\nu)} \right), \quad (18)$$

where the fields $\mathbf{E}^{(\nu)}$ and $\mathbf{B}^{(\nu)}$ are derived from the potential $A^{(\nu)} = (A_0^{(\nu)}, \mathbf{A}^{(\nu)})$ through

$$\mathbf{E}^{(\nu)} = -\nabla A_0^{(\nu)} - \frac{1}{c} \frac{\partial \mathbf{A}^{(\nu)}}{\partial t} \quad ; \quad \mathbf{B}^{(\nu)} = \nabla \times \mathbf{A}^{(\nu)} \quad . \quad (19)$$

Finally Eq.(16) can be written in a form where the contributions of the nonvortical and vortical stress potentials appear separately

$$\frac{\partial \mathcal{S}}{\partial t} + \frac{1}{2\mu} \left(\nabla \mathcal{S} - \frac{q}{c} \mathbf{A}^{(ex)} - \frac{q}{c} \mathbf{A}^{(\nu)} \right)^2 + q A_0^{(\varsigma)} + q A_0^{(ex)} + q A_0^{(\nu)} = 0 \quad . \quad (20)$$

We remark that Eq.s (20) and (15) are completely equivalent to Eq. (7).

The continuity equation, after taking into account (15), assumes the following form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left[\frac{1}{\mu} \left(\nabla \mathcal{S} - \frac{q}{c} \mathbf{A}^{(ex)} - \frac{q}{c} \mathbf{A}^{(\nu)} \right) \rho \right] = 0 \quad . \quad (21)$$

It is important to emphasize that Eq.s (20), (21) and (15) describe the most general hydrodynamics in an alternative fashion with respect to the usual one, which we find in the texts of fluid physics. Clearly there are still considerable degrees of freedom in this hydrodynamics and it necessitates some constitutive equations. It is perhaps remarkable that one can obtain these additional equations, as we will see in the following, within the theory and without making any extra assumption.

III. QUANTUM EVOLUTION EQUATION

Eq. (17) has a very transparent physical meaning simply imposing that the nonvortical stress force $\mathbf{F}^{(\varsigma)}$ is conservative. After introducing the field $\xi = \ln \rho$ we can write this equation under the form

$$q \frac{\partial A_0^{(\varsigma)}}{\partial x_j} - \frac{\partial \varsigma_{jk}}{\partial x_k} - \varsigma_{jk} \frac{\partial \xi}{\partial x_k} = 0 \quad . \quad (22)$$

Note that Eq. (22) is a condition constraining the forms of $A_0^{(\varsigma)}$ and ς_{jk} which can be viewed as two functionals of the field ξ . Even though it can appear that Eq. (22) contains considerable degrees of freedom, in the following we will show that the particular structure of this equation restricts strongly the number of its solutions.

We observe that just as the first term in Eq. (22) also the second and third term must take the form $\partial(\dots)/\partial x_j$. This requirement can be satisfied by posing $\varsigma_{jk} = \varsigma \delta_{jk}$. Note that this choice is compatible with the symmetry

property $\varsigma_{jk} = \varsigma_{kj}$ imposed by the monad kinetics and permits to write Eq. (22) as

$$q \frac{\partial A_0^{(\varsigma)}}{\partial x_j} - \frac{\partial \varsigma}{\partial x_j} - \varsigma \frac{\partial \xi}{\partial x_j} = 0 \quad . \quad (23)$$

The structure of the third term in Eq. (23) imposes that ς must be an arbitrary function of ξ so that we can write

$$\varsigma \frac{\partial \xi}{\partial x_j} = \frac{\partial}{\partial x_j} \int \varsigma d\xi \quad . \quad (24)$$

In this way Eq. (22) assumes the following simple form

$$\frac{\partial}{\partial x_j} \left(q A_0^{(\varsigma)} - \varsigma - \int \varsigma d\xi \right) = 0 \quad . \quad (25)$$

Eq. (25) allows to express $A_0^{(\varsigma)}$ in terms of ς , which remains an arbitrary function of the variable ρ . Then we can write the first solution (or class of solutions being $\varsigma(\rho)$ arbitrary) of Eq. (22) as

$$\varsigma_{jk} = \varsigma(\rho) \delta_{jk} \quad , \quad (26)$$

$$q A_0^{(\varsigma)} = \varsigma(\rho) + \int d\rho \frac{\varsigma(\rho)}{\rho} \quad . \quad (27)$$

This solution describes the classical Eulerian fluid with density of internal energy $\varepsilon = \varsigma_{jj}/2$ given by $\varepsilon = 3 \varsigma(\rho)/2$ and pressure $\pi = \rho \varsigma(\rho)$.

We show now that besides the above classical solution, Eq. (22) admits a second more interesting and less evident solution. Again, we recall that all the terms of Eq. (22) must take the form $\partial(\dots)/\partial x_j$. The particularly simple structure of the second term suggests to chose $\varsigma_{jk} = \partial a_k / \partial x_j$. Using the symmetry property $\varsigma_{jk} = \varsigma_{kj}$ we have that $a_k = \partial \alpha / \partial x_k$ and then the density of residual stress tensor assumes the form

$$\varsigma_{jk} = \frac{\partial^2 \alpha}{\partial x_j \partial x_k} \quad , \quad (28)$$

with α an unknown scalar functional depending on the field ξ . Eq. (22) can be written now as

$$\frac{\partial}{\partial x_j} \left(q A_0^{(\varsigma)} - \frac{\partial^2 \alpha}{\partial x_k \partial x_k} \right) - \frac{\partial^2 \alpha}{\partial x_j \partial x_k} \frac{\partial \xi}{\partial x_k} = 0 \quad . \quad (29)$$

By making the hypothesis that the functional α is a function of the field ξ (it can be easily verified that this is the only possibility), and after developing the derivatives of $\alpha(\xi)$ we obtain

$$\frac{\partial^2 \alpha}{\partial x_j \partial x_k} = \frac{d\alpha}{d\xi} \frac{\partial^2 \xi}{\partial x_j \partial x_k} + \frac{d^2 \alpha}{d\xi^2} \frac{\partial \xi}{\partial x_j} \frac{\partial \xi}{\partial x_k} \quad . \quad (30)$$

Consequently, the third term in Eq.(29) becomes

$$\begin{aligned} \frac{\partial^2 \alpha}{\partial x_j \partial x_k} \frac{\partial \xi}{\partial x_k} &= \frac{\partial}{\partial x_j} \left[\frac{1}{2} \frac{d\alpha}{d\xi} \frac{\partial \xi}{\partial x_k} \frac{\partial \xi}{\partial x_k} \right] \\ &+ \frac{1}{2} \frac{d^2 \alpha}{d\xi^2} \frac{\partial \xi}{\partial x_j} \frac{\partial \xi}{\partial x_k} \frac{\partial \xi}{\partial x_k} \quad . \quad (31) \end{aligned}$$

The requirement that also this third term has the form $\partial(\dots)/\partial x_j$, imposes that $d^2\alpha/d\xi^2 = 0$ so that Eq. (22) assumes the form

$$\frac{\partial}{\partial x_j} \left[qA_0^{(\varsigma)} - \frac{d\alpha}{d\xi} \frac{\partial^2 \xi}{\partial x_k \partial x_k} - \frac{1}{2} \frac{d\alpha}{d\xi} \frac{\partial \xi}{\partial x_k} \frac{\partial \xi}{\partial x_k} \right] = 0 \quad (32)$$

The condition $d^2\alpha/d\xi^2 = 0$ is a second order ordinary differential equation which can be trivially integrated providing $\alpha = c_0\xi + c_1$ with c_0 and c_1 arbitrary integration constants. The expressions of ς_{jk} and $A_0^{(\varsigma)}$, follow immediately from Eq.(28) and Eq. (32) respectively

$$\varsigma_{jk} = c_0 \frac{\partial^2}{\partial x_j \partial x_k} \ln \rho \quad , \quad (33)$$

$$qA_0^{(\varsigma)} = c_0 \left[\Delta \ln \rho + \frac{1}{2} (\nabla \ln \rho)^2 \right] \quad . \quad (34)$$

The constant c_1 , not influencing the values of ς_{jk} and $A_0^{(\varsigma)}$, can be set equal to zero.

From the definition of the total internal energy related to nonvortical flow we have $\mathcal{H} = \int d^3x \rho \varepsilon > 0$ being $\varepsilon = \varsigma_{jj}/2 > 0$ its density [4, 12]. Starting from Eq. (33) we can easily calculate \mathcal{H} obtaining

$$\mathcal{H} = \frac{\eta^2}{8\mu} I \quad ; \quad I = \int d^3x \frac{1}{\rho} (\nabla \rho)^2 \quad , \quad (35)$$

where we have posed $c_0 = -\eta^2/4\mu < 0$ in order to have $\mathcal{H} > 0$. The real positive constant η remains a free parameter of the theory. It is remarkable that \mathcal{H} turns out to be proportional to the Fisher information measure I (Fisher 1922) [3, 13] which in this way, emerges in quantum mechanics naturally.

Note that after setting $\eta = \hbar/N$ the quantity $\epsilon A_0^{(\varsigma)}$ given by Eq. (34) results to be the quantum potential (Madelung 1926) [1, 2]. In this way, the fundamental constant \hbar comes out simply as an integration constant, while the quantum potential emerges naturally as one of the two possible solutions of Eq. (22).

The two solutions above obtained are the only possible ones. At the present, we do not have any criterion to judge whether one solution or the other is the right solution to be chosen as constitutive equation for the system. It is trivial to verify that the most general solution of Eq. (22) can be expressed as a linear combination of these independent solutions and assumes the form

$$\varsigma_{jk} = \delta_{jk} \frac{1}{\rho} \int \rho \frac{dU(\rho)}{d\rho} d\rho - \frac{\hbar^2}{4m} \frac{\partial^2}{\partial x_j \partial x_k} \ln \rho \quad , \quad (36)$$

$$\epsilon A_0^{(\varsigma)} = U(\rho) - \frac{\hbar^2}{4m} \left[\Delta \ln \rho + \frac{1}{2} (\nabla \ln \rho)^2 \right] \quad , \quad (37)$$

being $U(\rho)$ an arbitrary function. Eq. (37) represents the wanted constitutive equation and has been enforced exclusively from the fact that the nonvortical component of stress force is conservative.

Note that the constitutive equation (37), which gives the potential $\epsilon A_0^{(\varsigma)}$ starting from ρ , has been obtained in the framework of an entirely classical kinetics. This kinetics describes the subquantum statistical ensemble of the N interacting monads. The monad interaction, which is not specified, generates the stress forces and then the potential $\epsilon A_0^{(\varsigma)}$. Clearly only when the monad interactions are suppressed the potential vanishes. Then the presence of the collision integral $C(f)$ in the Eq. (2) is necessary for the consistency of the theory. Concerning the potential $\epsilon A_0^{(\varsigma)}$ we note that in the limit $\hbar \rightarrow 0$, becomes $\epsilon A_0^{(\varsigma)} = U(\rho)$. In this case the constitutive equation describes a classical fluid which is governed by Eqs (20) and (21).

Observe that the system can be described through the two real fields ρ and \mathcal{S} , whose evolution equations are (21) and (20), respectively. Obviously, these equations must be considered together with the constitutive equation (37) that defines $A_0^{(\varsigma)}$. Alternatively one can describe the system by means of the complex field $\Psi = \rho^{1/2} \exp(iS/\hbar)$ with $S = N\mathcal{S}$. The evolution equation for the field Ψ can be obtained trivially by using the standard procedure [4], starting from Eqs (20), (21) and (37)

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m} \left(-i\hbar \nabla - \frac{e}{c} \mathbf{A}^{(ex)} - \frac{e}{c} \mathbf{A}^{(\nu)} \right)^2 \Psi + U(\rho) \Psi + \epsilon A_0^{(ex)} \Psi + \epsilon A_0^{(\nu)} \Psi \quad . \quad (38)$$

Eq. (38) can describe the non relativistic quantum particle with spin in an external electromagnetic field generated from the potential $A^{(ex)} = (A_0^{(ex)}, \mathbf{A}^{(ex)})$. The second potential $A^{(\nu)} = (A_0^{(\nu)}, \mathbf{A}^{(\nu)})$ represents internal degrees of freedom and describes the vortical flow of the system with $\omega \neq 0$. Finally in Eq. (38) a third, internal, arbitrary and nonlinear potential namely $U(\rho)$ appears, which can be exploited to investigate some complex phenomenologies of the condensed matter arising from collective interactions (e.g. the Bose-Einstein condensation has been studied previously in the literature by considering $U(\rho) = a\rho$).

Besides the equation of motion the present theory is characterized by the presence of a subsidiary condition restricting the velocity circulation. To show this we rewrite Eq.(15) under the form

$$m \mathbf{u} + \frac{e}{c} \mathbf{A}^{(ex)} + \frac{e}{c} \mathbf{A}^{(\sigma)} = \nabla S \quad . \quad (39)$$

Consider an arbitrary closed contour C delimiting the surface s , it is trivial to verify that from Eq. (39) follows

$$m \oint_C \mathbf{u} d\mathbf{l} + \frac{e}{c} \Phi - \frac{\hbar}{2} \int_s T_N ds = \Gamma \quad , \quad (40)$$

being $\Phi = \int_s B_N^{(ex)} ds$ the flux of the external magnetic field $\mathbf{B}^{(ex)}$ going through s , while the vector \mathbf{T} is proportional to the vorticity namely $\omega = (\hbar/2)\mathbf{T}$. Then if

we take into account that $\boldsymbol{\omega} = -(\epsilon/c)\mathbf{B}^{(\nu)}$ we have that the third term in the left hand side of Eq. (40) is proportional to the flux of the $\mathbf{B}^{(\nu)}$ going through s . Finally the quantity Γ is given by

$$\Gamma = \oint_C \nabla S d\mathbf{l} = \oint_C dS, \quad (41)$$

and its value does not depend on the detailed path of C in so far as it does not pass through a singular line because $\nabla \times \nabla S = 0$ elsewhere. We remark that the condition (40) holds independently on the spin value which clearly influences the value of Γ . For spinless particles ($\mathbf{T} = 0$) it has been shown that $\Gamma = n\hbar$ with n an arbitrary integer [4, 5]. The case of a non relativistic particle with spin 1/2 has been considered extensively in the framework of the Schrödinger-Pauli theory, by several authors [5, 6, 7, 8], and it has been shown that $\Gamma = n\hbar/2$ [5].

IV. CONCLUDING REMARKS

We tackle briefly the problem concerning the locality in quantum physics, that was left unresolved in the twenty three year long debate between Einstein and Bohr and was reconsidered by Bell in 1964. It is well known that the Bell's inequality has been obtained in the framework of local, hidden variables and deterministic theories. This inequality is in disagreement both with quantum mechanics and experimental evidence. The reason of this disagreement now appears clear. Here we have obtained quantum mechanics starting from the underlying monad kinetics which is a non local, hidden variables and probabilistic theory. Indeed, in the proposed scenario, the quantum particle of mass m turns out to be a statistical system, having a spatial extension and an internal structure. It is composed by N identical subquantum interacting particles of mass μ , the monads. These monads obey the laws of classical physics and their dynamics is described in the phase space by the standard kinetic equation.

Remark that we don't make any assumption about the structure of the collision integral and the nature of the interaction between the monads except that, during the point-like collisions the monad number, momentum and energy are conserved. The projection of the above phase space kinetics into the physical space produces a hydrodynamics which leads naturally to the one particle quantum mechanics. On the other hand the monad interaction generates the stress forces and then the quantum potential which vanishes only when the monad interactions are suppressed. Then the presence of the collision integral in the kinetic Eq. (1) is necessary for the consistency of the theory. The determination of the form of the collision integral in the framework of a subquantum dynamics is a very interesting problem which remains still open.

At this point, the question arises spontaneously if it is possible to include the locality in quantum physics. Clearly, there is no problem in principle to construct a theory starting from a relativistic kinetic equation rather than from Eq. (2). After noting that a subquantum relativistic kinetics is a local, hidden variables and probabilistic theory, we can make the conjecture that *a subquantum relativistic monad kinetics could be underlying a local quantum theory*.

To summarize we have shown that the approach to quantum mechanics proposed in ref. [4] is suitable to treat also the quantum particle with spin as well as the nonlinear quantum mechanics. Main goal of the present paper is the quantum evolution equation (38) which has been obtained within an entirely classical subquantum kinetics without making any extra assumption. This equation: i) can describe the quantum particle with spin, due to the presence of the internal potential $A^{(\nu)}$; ii) can be used to study some complex phenomenologies in condensed matter physics e.g. Bose-Einstein condensation, due to the presence of the nonlinearity $U(\rho)$. Clearly when $A^{(\nu)} = 0$ and $U(\rho) = 0$, Eq. (38) reproduces the Schrödinger equation describing the spinless particle.

-
- [1] E. Madelung, Z. Phys. **40**, 322 (1926).
 - [2] D. Bohm, Phys. Rev. **85**, 166 (1952).
 - [3] M. Reginatto, Phys. Rev. A **58**, 1775 (1998).
 - [4] G. Kaniadakis, Physica A **307**, 172 (2002).
 - [5] T. Takabayasi, Prog. Theor. Phys. **8**, 143 (1952); **9**, 187 (1953); **70**, 1 (1983).
 - [6] M. Schönberg, Nuovo Cimento **XI** **6**, 674 (1954).
 - [7] R. Schiller, Phys. Rev. **125**, 1116 (1962).
 - [8] S. K. Ghosh and B. M. Deb, Phys. Rep. **92**, 1 (1982).
 - [9] T. Waite, A.O. Barut, and J.R. Zeni, *Electron Theory and Quantum Electrodynamics: 100 years later*, Editor A. Dowling, (Plenum Press, New York, 1997).
 - [10] G. Kaniadakis and A.M. Scarfone, J. Phys. A: Math. Gen. **35**, 1943 (2002).
 - [11] R. L. Liboff, *Kinetics Theory*, (Prentice-Hall International Editors, New Jersey, 1990).
 - [12] R. T. Glassey, *The Cauchy problem in Kinetic Theory*, (SIAM, Philadelphia, PA, 1996).
 - [13] B.R. Frieden and B.H. Soffer, Phys. Rev. **52**, 2274 (1995).